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DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

37. **Proposed by A. H. BELL, Box 184, Hillsboro, Illinois.**

Find the first four integral values of n in $\frac{n(5n-3)}{2}=\square$.

I. **Solution by the PROPOSER, and Prof. J. SCHEFFER, A. M., Hagerstown, Maryland.**

Let the heptagonal numbers $\frac{n(5n-3)}{2}=\square=y^2$. Clearing of fractions, then multiplying by 20 and adding 9 to both sides, $(10n-3)^2=40y^2+9=\square=x^2$. $\therefore n=(x+3)/10$........(1). Let $x^2-40y^2=9$ be written $3^2x_1{}^2-40.3^2y_1{}^2=3^2$. Dividing by 3^2 and solving $x_1{}^2-40y_1{}^2=1$, the convergent of $\sqrt{40}$ is $19/3$. $\therefore x_1=19$; by the general formula $x_{n+1}=2x_1\times x_n-x_{n-1}$, we have $x_1=1$, 19, 721, 27379, 1039681, 39, 480, 499, etc. As $x=3x_1$ and as integral values for n can only be obtained by the numbers ending in 9, then in (1) $n=1$, 6, 8214, and 11844150.

II. **Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.**

The expression readily reduces to $10n^2-6n=\square$......(1). It is readily seen that $n=1$ satisfies this equation. Take $n=m+1$, substitute it in (1), reduce and we have $10m^2+14m+4=\square=(\text{say})\ (pm-2)^2$, from which we obtain $m=(4p+14)/(p^2-10)$. Take $p=4$ and we have $m=5$, and $n=6$, the second value. Now take $n=m+6$, substitute in (1) and reduce as before and we find, $m=43$, and $n=49$, the third value. In $(4p+14)/(p^2-10)$ take $p=19/6$, $p^2-10=1/36$ and we have $m=960$, and $n=961$, the fourth value.

III. **Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.**

If we put the expression equal x^2 and reduce, we readily obtain $10n=3\pm\sqrt{40x^2+9}$. Putting $x=1$, 2, 9, 40 and 77, respectively, I find the first four integral values of n to be, respectively, ±1, 6, -25, and 49.

38. **Proposed by H. C. WILKES, Skull Run, West Virginia.**

Let n be any number and let $n^3+1=x$.
Then $x^3+(2x-3)^3+(nx-3n)^3=n^3x^3$. Demonstrate.

I. **Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.**

The simplest way is to substitute the value of x and expand. An identity is the result.

II. **Solution by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.**

Substituting n^3+1 for x, $(n^3+1)^3+(2n^3-1)^3+(n^4-2n)^3=(n^4+n)^3$, which, if we put c for n, is the same as equation (12) on page 155 of Vol. II., No. 9 of the *Mathematical Magazine*, and is an identity as will be found by performing the indicated operations and adding.

III. Solution by BENJ. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, Ohio.

Suppose the statement true : $x^3+(2x-3)^3+(nx-3n)^3=n^3x^3$.

Then, $x^3+8x^3-36x^2+54x-27+n^3x^3-9n^3x^2+27n^3x-27n^3=n^3x^3$.

Whence, $(x-1)(x^2-3x+3)-n^3(x^2-3x+3)=0$. Whence, $x-1-n^3=0$.

Whence, $n^3+1=x$, which is the hypothesis. ∴ The above supposition is true.

IV. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics in the University of Tennessee, Knoxville, Tennessee.

Performing the operations we have

$$x^3+8x^3-36x^2+54x-27+n^3x^3-9n^3x^2+27n^3x-27n^3, \text{ or}$$

$$9x^3-36x^2+54x-27+n^3x^3-9n^3x^2+27n^3(x-1), \text{ but } n^3=x-1\ ;$$

hence $9x^3-36x^2+54x-27+n^3x^3-9n^3x^2+27(x-1)^2$, which upon reduction gives

$$9x^3-9x^2-9n^3x^2+n^3x^3,=9x^3-9x^2-9x^2(x-1)+n^3x^3,$$

$$=9x^3-9x^2-9x^3+9x^2+n^3x_3,\ =n_3x_3.$$

Also solved by *J. H. DRUMMOND*, *M. A. GRUBER*, *J. SCHEFFER*, and *G. B. M. ZERR*.

39. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

The m^{th} root of the n^{th} power of an *integral* number is a perfect p^{th} power. What is the number ?

Solutions by J. H. DRUMMOND, LL. D., Portland, Maine ; M. A. GRUBER, A. M., Washington, D. C., and J. SCHEFFER, A. M., Hagerstown, Maryland.

Let $x^{1\div m}=a^p$, then $x=a^{pm}$, in which a may be any integral number : for the n^{th} power of a^{pm} must also be a p^{th} power. [J. H. DRUMMOND.]

Manifestly any whole number raised to the mp^{th} power.

[J. SCHEFFER.]

Let x=the integral number. Then $x^{n\div m}=a^p$. Raising to m^{th} power and extracting n^{th} root, we obtain $x=a^{mp\div n}$, or $\sqrt[n]{a^{mp}}$. ∴ The required integral number is the n^{th} root of the mp^{th} power of any integer, mp being a multiple of n.

[M. A. GRUBER.]

Also solved by *G. B. M. ZERR*.